Fermion symmetry flavors

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Abstract

Elementary fermions can be represented by couplings of two quaternionic fields. Each of these fields can be represented by a pair of a quaternionic function and a quaternionic parameter space. The parameter spaces and the functions differ in their symmetry flavor. The reverse bra-ket method can be used to relate these fields, the corresponding functions and their parameter spaces to operators that reside in quaternionic Hilbert spaces. The eigenspaces of these operators act as structured storage places. Obviously the properties of the elementary fermions and their behavior are directly related to the symmetry flavors of the coupled fields.

Introduction

Quaternionic number systems exist in many versions that differ in the way that these number systems are ordered. For example it is possible to order the real parts of the quaternions up or down. Or a Cartesian coordinate system can be used to order the imaginary parts of the quaternions. This can be done on eight mutually independent ways. It is also possible to apply spherical symmetric ordering by using a spherical coordinate system. This can be done by starting with the azimuth and order it up or down and then order the polar angle and order it up or down. It is also possible to start with the polar angle. A spherical coordinate system starts from a selected Cartesian coordinate system.

The reverse bra-ket method [1] enables to attach all these different symmetry flavors of the quaternionic number system to dedicated operators that reside in an infinite dimensional separable quaternionic Hilbert space. Separable Hilbert spaces can only handle countable eigenspaces. Thus the reverse bracket method can only use the rational subsets of the quaternionic number systems.

Each infinite dimensional separable Hilbert space owns a companion Gelfand triple, which is a non-separable Hilbert space and which also supports operators that feature continuums as their eigenspaces. The reverse bra-ket method relates operators in the separable Hilbert space o operators in the Gelfand triple.

These representations of quaternionic number systems can act as parameter spaces of quaternionic functions that can also be represented by operators and their eigenspaces. The reverse bra-ket method establishes this link.

Together, this means that the two companion quaternionic Hilbert spaces can represent fields via the eigenspaces of some of their operators and that these fields can also be represented by pairs of quaternionic functions and their parameter spaces.

Reverse bra-ket method

The reverse bra-ket method enables the definition of several different parameter spaces that can coexist in the same quaternionic Hilbert space.

Let $\{q_i\}$ be the set of rational quaternions in a selected quaternionic number system and let $\{|q_i\rangle\}$ be the set of corresponding base vectors. They are eigenvectors of the normal operator $\mathcal{R}$. We enumerate the base vectors with index i.

$$\mathcal{R} \equiv |q_i\rangle q_i \langle q_i| \tag{1}$$

$\mathcal{R}$ is the configuration parameter space operator.

This notation must not be interpreted as a simple outer product between a ket vector $|q_i\rangle$, a quaternion q_i and a bra vector $\langle q_i|$. It involves a complete set of eigenvalues $\{q_i\}$ and a complete orthomodular set of Hilbert vectors $\{|q_i\rangle\}$. It implies a summation over these constituents, such that for all bra's $\langle x|\rangle$ and ket's $|y\rangle$:

$$\langle x|\mathcal{R}\, y\rangle = \sum_i \langle x|q_i\rangle q_i \langle q_i|y\rangle \tag{2}$$

$\mathcal{R}_0 = (\mathcal{R} + \mathcal{R}^\dagger)/2$ is a self-adjoint operator. Its eigenvalues can be used to arrange the order of the eigenvectors by enumerating them with the eigenvalues. The ordered eigenvalues can be interpreted as ***progression values***.

$\boldsymbol{\mathcal{R}} = (\mathcal{R} - \mathcal{R}^\dagger)/2$ is an imaginary operator. Its eigenvalues can also be used to order the eigenvectors. The eigenvalues can be interpreted as ***spatial values*** and can be ordered in several ways.

Similarly in the Gelfand triple a corresponding reference operator $\mathfrak{R}$ can be defined.

$$\mathfrak{R} \equiv |q\rangle q \langle q| \tag{3}$$

For all bra's $\langle x|\rangle$ and ket's $|y\rangle$ holds:

$$\langle x|\mathfrak{R}\, y\rangle = \int_q \langle x|q\rangle q \langle q|y\rangle \; dq \tag{4}$$

The reverse bra-ket method relates pairs of natural parameter spaces and quaternionic functions to operators and their eigenspaces. $\mathcal{F}$ defines a new operator that is based on quaternionic function $\mathcal{F}(q)$. Here we suppose that the target values of $\mathcal{F}$ belong to the same version of the quaternionic number system as its parameter space does. Operator $\mathcal{F}$ has a continuum quaternionic eigenspace.

$$\mathcal{F} \equiv |q\rangle \mathcal{F}(q) \langle q| \tag{5}$$

This is a shorthand for:

$$\langle x|\mathcal{F}\, y\rangle = \int_q \langle x|q\rangle \mathcal{F}(q) \langle q|y\rangle \; dq \tag{6}$$

The same trick works in the separable Hilbert space.

$$f \equiv |q_i\rangle f(q_i)\langle q_i| \tag{7}$$

f defines a new operator that is based on quaternionic function $f(q)$. Here we suppose that the target values of f belong to the same version of the quaternionic number system as its parameter space does. Operator f has a countable set of discrete quaternionic eigenvalues.

For this operator the reverse bra-ket notation is a shorthand for

$$\langle x|f\, y\rangle = \sum_i \langle x|q_i\rangle f(q_i)\langle q_i|y\rangle \tag{8}$$

Symmetry centers

Symmetry centers are representatives of parameter spaces that contain the spherically ordered version of the imaginary part of a quaternionic number system. Symmetry centers can float with respect to a background parameter space that is formed by a complete quaternionic number system, which features Cartesian ordering. The background parameter space is well ordered. It means that its members can be enumerated by its real parts. As a consequence that real part can be interpreted as progression. The background parameter space is the eigenspace of the normal reference operator $\mathcal{R}$.

Symmetry centers have a well-defined spatial origin. That origin floats on the eigenspace of the reference operator $\mathcal{R}$. Symmetry centers are formed by a dedicated category of ***compact anti-Hermitian operators*** $\{\mathfrak{S}_n^x\}_n$.

Each symmetry center corresponds to a dedicated subspace of the infinite dimensional separable Hilbert space. That subspace is spanned by the eigenvectors $\{|\mathfrak{s}_i^x\rangle\}$ of a corresponding symmetry center reference operator $\mathfrak{S}_n^x$. Here the superscript ^{x} refers to the type of the symmetry center. The subscript n enumerates the symmetry centers. The type covers more than just the symmetry flavor. We will often omit the subscript.

An infinite dimensional separable Hilbert space can house a set of subspaces that each represent such a symmetry center. Each of these subspaces then corresponds to a dedicated spherically ordered reference operator $\mathfrak{S}_n^x$. The superscript ^{x} distinguishes between symmetry flavors and other properties, such as spin and generation flavor. Symmetry centers correspond to dedicated subspaces that are spanned by the eigenvectors $\{|\mathfrak{s}_i^x\rangle\}$ of the symmetry center reference operator $\mathfrak{S}^x$. (Here we omit subscript n).

$$\mathfrak{S}^x = |\mathfrak{s}_i^x\rangle \mathfrak{s}_i^x \langle \mathfrak{s}_i^x| \tag{1}$$

$$\mathfrak{S}^{x\dagger} = -\mathfrak{S}^x \tag{2}$$

Only the location of the center inside the eigenspace of reference operator $\mathcal{R}$ is a progression dependent value. This value is not eigenvalue of operator $\mathfrak{S}_n^x$. The location of the center inside $\mathcal{R}^{⓪}$ is eigenvalue of a central governance operator $\mathcal{g}$.

The closed subspaces that correspond to a symmetry center have a fixed finite dimension. This dimension is the same for all types of symmetry centers. This ensures that symmetry related charges all appear in the same short list.

Symmetry flavors

Quaternions can be mapped to Cartesian coordinates along the orthonormal base vectors 1, $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$; with $\boldsymbol{ij} = \boldsymbol{k}$

Due to the four dimensions of quaternions, quaternionic number systems exist in 16 well-ordered versions $\{q^x\}$ that differ only in their discrete Cartesian symmetry set. The quaternionic number systems $\{q^x\}$ correspond to 16 versions $\{q_i^x\}$ of rational quaternions.

Half of these versions are right handed and the other half are left handed. Thus the handedness is influenced by the symmetry flavor.

The superscript ^{x} can be ⓪, ①, ②, ③, ④, ⑤, ⑥, ⑦, ⑧, ⑨, ⑩, ⑪, ⑫, ⑬, ⑭, or ⑮.

This superscript represents the ***symmetry flavor*** of the superscripted subject. For the reference operator we neglect the superscript ⓪.

The reference operator $\mathcal{R} = |q_i\rangle q_i \langle q_i|$ in separable Hilbert space $\mathfrak{H}$ maps into the reference operator $\mathfrak{R} = |q\rangle q \langle q|$ in Gelfand triple $\mathcal{H}$.

The symmetry flavor of the symmetry center $\mathfrak{S}^x$, which is maintained by operator $\mathfrak{S}^x = |\mathfrak{s}_i^x\rangle \mathfrak{s}_i^x \langle \mathfrak{s}_i^x|$ is determined by its Cartesian ordering and then compared with the reference symmetry flavor, which is the symmetry flavor of the reference operator $\mathcal{R}$. ↑↑↑↑

Now the symmetry related charge follows in three steps.

1. Count the difference of the spatial part of the symmetry flavor of symmetry center $\mathfrak{S}^x$ with the spatial part of the symmetry flavor of reference operator $\mathcal{R}$.
2. If the handedness changes from **R** to **L**, then switch the sign of the count.
3. Switch the sign of the result for anti-particles.

We use the names of the corresponding particles that appear in the standard model in order to distinguish the different symmetry flavor combinations. Elementary fermions relate to solutions of a corresponding second order partial differential equation that describes the embedding of these particles. Elementary bosons relate to solutions of a different second order partial differential equation.

Fermion symmetry flavor					
Ordering x y z τ	Super script	Handedness Right/Left	Color charge	Electric charge * 3	Symmetry center type. Names are taken from the standard model
↑↑↑↑	⓪	**R**	N	+0	neutrino
↓↑↑↑	①	**L**	R	−1	down quark
↑↓↑↑	②	**L**	G	−1	down quark
↓↓↑↑	③	**L**	B	−1	down quark
↑↑↓↑	④	**R**	B	+2	up quark
↓↑↓↑	⑤	**R**	G	+2	up quark
↑↓↓↑	⑥	**R**	R	+2	up quark
↓↓↓↑	⑦	**L**	N	−3	electron
↑↑↑↓	⑧	**R**	N	+3	positron
↓↑↑↓	⑨	**L**	R	−2	anti-up quark

↑↓↑↓	⑩	**L**	G	−2	anti-up quark
↓↓↑↓	⑪	**L**	B	−2	anti-up quark
↑↑↓↓	⑫	**R**	B	+1	anti-down quark
↓↑↓↓	⑬	**R**	R	+1	anti-down quark
↑↓↓↓	⑭	**R**	G	+1	anti-down quark
↓↓↓↓	⑮	**L**	N	−0	anti-neutrino

Elementary fermions switch their handedness when the sign of the real part is switched. Spherical ordering can be done by first starting with the azimuth and next proceeding by the polar angle. Both can be done up or down. Fermions and bosons appear to differ in this choice.

Also continuous functions and continuums feature a symmetry flavor. Continuous quaternionic functions $\psi^x(q^x)$ and corresponding continuums do not switch to other symmetry flavors ^{y}.

The reference symmetry flavor $\psi^y(q^y)$ of a continuous function $\psi^x(q^y)$ is the symmetry flavor of the parameter space $\{q^y\}$.

If the continuous quaternionic function describes the density distribution of a set $\{a_i^x\}$ of discrete objects a_i^x, then this set must be attributed with the same symmetry flavor ^{x}. The real part describes the location density distribution and the imaginary part describes the displacement density distribution.

Fields

Symmetry centers feature a ***symmetry related charge*** that depends on the difference between the symmetry flavor of the symmetry center and the symmetry flavor of the reference operator $\mathcal{R}$, which equals the symmetry flavor of the embedding continuum $\mathfrak{C}$. The symmetry related charges raise a ***symmetry related field*** $\mathfrak{A}$. The symmetry related field $\mathfrak{A}$ influences the position of the center of the symmetry center in parameter space $\mathcal{R}$ and indirectly it influences the position of the map of the symmetry center into the field that represents the embedding continuum $\mathfrak{C}$. Both fields, $\mathfrak{A}$ and $\mathfrak{C}$ use the eigenspace of the reference operator $\mathfrak{R}$ as their parameter space.

References

[1] J.A.J. van Leunen. (2015), The Reverse Bra-Ket Method. http://vixra.org/abs/1511.0266 .

[2] J.A.J. van Leunen. (2015), On the Origins of Physical Fields. http://vixra.org/abs/1511.0007 .